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# Optimal allocation of a heat-exchanger inventory in heat driven refrigerators

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Abstract—This paper reports the thermodynamic optimization (or entropy generation minimization) of a heat-driven refrigeration plant, that is, a refrigerator without work input, which is driven by a heat source. The treatment accounts for the heat transfer irreversibilities of the three heat exchangers, and for the finiteness of the total heat-exchanger inventory. The operating conditions for maximum refrigeration rate are determined. It is shown that the heat-exchanger inventory must be divided optimally between the three heat exchangers. For example, half of the inventory must be placed in the heat exchanger used to reject heat to the ambient. The maximum refrigeration rate per unit of total heat exchanger inventory is reported. These thermodynamic optimization principles are then applied to a refrigerator driven by heat transfer from a solar collector.

#### **1. INTRODUCTION**

The method of entropy generation minimization has emerged during the last two decades as a distinct subfield in heat transfer (e.g. refs. [1, 2]). The method consists of the simultaneous application of heat transfer and thermodynamics principles in the pursuit of realistic models of heat transfer processes, devices and installations. By 'realistic' models we mean models that account for the inherent irreversibility of heat, mass and fluid flow processes. In engineering, the entropy generation minimization method is also known as thermodynamic optimization and thermodynamic design.

The importance and growth of this field are further illustrated by the emergence of a parallel activity in physics. The physics work is usually referred to as thermodynamics in finite time (e.g. ref. [3]), and its methodology is the same combination of heat transfer and thermodynamics. Some of the most fundamental results refer to the optimization of power plants and refrigeration plants with heat transfer irreversibilities. In the power generation area, the focus has been on the regime for the production of maximum instantaneous power [4–7], which is equivalent to the regime of minimum entropy generation rate (cf. the Gouy-Stodola theorem, ref. [1], p. 24).

In the refrigeration area, the models that have been optimized based on this method had power input and

heat rejection to the ambient [8], as in the case of the vapor compression cycle [9]. They were optimized by maximizing the refrigeration load (rate of heat extraction from the cold space), which is the same as minimizing the rate of entropy generation of the refrigeration plant.

In this paper we apply the method of thermodynamic optimization to a distinct class of refrigeration plants: heat driven refrigeration plants, or plants without work input (Fig. 1, left). Examples of such plants are absorption refrigerators (e.g. ref. [10]), and jet ejector refrigerators (e.g. ref. [11]). Our objectives are to determine:

- 1. The operating conditions for maximum refrigeration effect, and
- 2. The optimal way of dividing a finite supply of heat exchanger surface between the three heat exchangers of the refrigeration plant.

Heat driven refrigerators constitute an important class, fundamentally, because of the peculiarity of almost no work input, and, practically, because the driving heat transfer ( $Q_H$  in Fig. 1) can be low-grade heat (e.g. solar [12]) or waste heat. The utilization of low-grade heat sources is stressed by environmental and economic considerations. For this reason we conclude the paper with an application of the present method to the optimization of a refrigerator driven by heat transfer from a solar collector.

a, b	constants	x conductance fraction, equation (19)
A	heat transfer area	<i>y</i> conductance fraction, equation (29)
$A_{\rm C}$	collector area	z conductance fraction, equation (29).
В G <sub>T</sub> р <sub>н</sub> р <sub>L</sub> Р	dimensionless group, equation (35) irradiance on collector surface price of heat input price of refrigeration load profit function, equation (26)	Greek symbols $\tau$ dimensionless temperature, $T/T_0$ .
$Q_{\rm H}$	heat input	Subscripts and superscripts
$\tilde{Q}_{\rm L}$	refrigeration load	() <sub>c</sub> Carnot (reversible) compartment
$\bar{2}_0$	condenser heat transfer	$()_{\rm H}$ generator, or solar collector
T <sub>H</sub>	generator temperature	() <sub>L</sub> evaporator
$T_{L}$	refrigeration load temperature	() <sub>max</sub> maximum
$T_{\rm st}$	collector stagnation temperature	() <sub>opt</sub> optimum
$T_0$	ambient temperature	$()_{P}, ()^{P}$ power plant part
U	overall heat transfer coefficient based on A	$()_{R}, ()^{R}$ refrigeration plant part () <sub>0</sub> ambient
W	power output	$(\tilde{})$ dimensionless variable.

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#### 2. MODEL WITH THREE HEAT TRANSFER **IRREVERSIBILITIES**

The main external features of a heat driven refrigerator are shown on the left side of Fig. 1. The working fluid executes cycles while removing the refrigeration load  $Q_{\rm L}$  from the refrigerated space  $T_{\rm L}$ , and rejecting the heat transfer  $Q_0$  to the ambient  $T_0$ . The cycle is driven by the heat transfer  $Q_{\rm H}$  received from the source temperature  $T_{\rm H}$ . There is no work transfer between the refrigerator and its environment.

From the outset, we recognize that the refrigerator operates irreversibly because of several entropy-generating mechanisms that are always present, for exam-

ple, heat transfer, throttling and mixing [11]. In the model shown on the right side of Fig. 1, we have divided the refrigerator into four compartments, the three heat exchangers  $(Q_{\rm H}, Q_0, Q_{\rm L})$  and the rest. The heat exchangers account for the irreversibility of the machine, and the remaining components (labeled (C)) are modeled as irreversibility free. In other words, in the following analysis we neglect the irreversibility associated with frictional pressure drops, throttling and mixing. This assumption is consistent with the thermodynamics treatment of power plants [4-7, 13] and refrigeration plants based on the vapor compression cycle [9].

The four compartments of the irreversible refriger-



Fig. 1. Model with heat transfer irreversibilities (right) for an irreversible heat-driven refrigerator (left).

ator model are described analytically by the four statements :

$$Q_{\rm H} = (UA)_{\rm H}(T_{\rm H} - T_{\rm HC}) \tag{1}$$

$$Q_0 = (UA)_0 (T_{0C} - T_0)$$
(2)

$$Q_{\rm L} = (UA)_{\rm L} (T_{\rm L} - T_{\rm LC}) \tag{3}$$

$$\frac{Q_{\rm H}}{T_{\rm HC}} + \frac{Q_{\rm L}}{T_{\rm LC}} = \frac{Q_0}{T_{\rm 0C}}.$$
 (4)

In adition, the first law of thermodynamics requires that

$$Q_{\rm H} + Q_{\rm L} = Q_0. \tag{5}$$

Each factor of type UA in equations (1)–(3) represents the overall thermal conductance of the respective heat exchanger, or the product between the heat transfer area A and the overall heat transfer coefficient U based on A. The thermal conductances  $(UA)_{\rm H}$ ,  $(UA)_0$  and  $(UA)_{\rm L}$  are rough measures of the sizes (amounts of hardware) of the three heat exchangers, therefore a reasonable economic constraint is [13, 14]

$$UA = (UA)_{\rm H} + (UA)_0 + (UA)_{\rm L}$$
, (constant) (6)

which states that the total thermal conductance inventory UA is fixed.

The question on which we focus in the following analysis is how to divide the UA inventory between  $(UA)_{\rm H}$ ,  $(UA)_0$  and  $(UA)_{\rm L}$ , such that the refrigeration rate  $Q_{\rm L}$  is maximized. This problem can be solved in several ways. One way is to use the system (1)-(5) to express  $Q_{\rm L}$  as a function of the thermal conductance distribution ratios  $(UA)_{\rm H}/UA$  and  $(UA)_{\rm L}/UA$ , and to maximize  $Q_{\rm L}$  with respect to these two degrees of freedom. This approach seems direct; however, the double maximization of  $Q_{\rm L}$  must be performed numerically and the competition between the three irreversibilities is hidden from view. Much more instructive is the alternative method shown next, which is based on conclusions known from the optimization of simpler power and refrigeration plants and takes us close to the final answer by using pure analysis.

## 3. COUPLED REFRIGERATION AND POWER PLANTS

The absorption refrigeration plant of Fig. 1 (right) is equivalent to the coupling of a power plant and refrigeration plant, as is shown in Fig. 2. The power output (W) from the power plant (P) drives the refrigeration plant (R). The spaces labeled (P) and (R) are irreversibility free. The heat rejection to the ambient, which in Fig. 1 was accommodated by a single heat exchanger,  $(UA)_0$ , is now effected by two heat exchangers operating in parallel,  $(UA)_0^P$  and  $(UA)_0^R$ . The equivalence between Figs. 1 and 2 is assured by writing

$$Q_0 = Q_0^{\rm P} + Q_0^{\rm R} \tag{7}$$



Fig. 2. The model of Fig. 1 (right) as the coupling of a heat transfer-irreversible power plant (P) with a heat transferirreversible refrigeration plant (R).

$$(UA)_0 = (UA)_0^{\rm P} + (UA)_0^{\rm R}.$$
 (8)

The power plant portion of the Fig. 2 model has been optimized in refs. [13, 14], in which the instantaneous power output W was maximized with respect to two degrees of freedom ( $T_{\rm HC}$  and  $T_{\rm 0C}$ , or  $T_{\rm HC}/T_{\rm 0C}$ and  $(UA)_{\rm H}/(UA)_{\rm 0}^{\rm P}$ ). The results that are relevant to the present study are

$$(UA)_{\mathrm{H,opt}} = (UA)_{0,\mathrm{opt}}^{\mathrm{P}}$$
(9)

$$W_{\rm max} = \frac{1}{4} (UA)_{\rm P} T_0(\tau_{\rm H}^{1/2} - 1)$$
 (10)

where  $(UA)_{\rm P}$  is the total thermal conductance inventory of the power-plant portion (P),

$$(UA)_{\rm P} = (UA)_{\rm H} + (UA)_0^{\rm P}$$
(11)

and

$$\tau_{\rm H} = \frac{T_{\rm H}}{T_0} > 1.$$
 (12)

Equation (9) states that  $(UA)_{\rm P}$  must be divided equally between the two heat exchangers of the power-plant portion. The question that remains is how much of the total inventory UA (Fig. 1) should be allocated to  $(UA)_{\rm P}$ .

Similar progress has been made in connection with the refrigerator portion (R) of the model of Fig. 2. Without repeating the analysis of refs. [8] and [15], we know that when W is given the refrigeration load  $Q_L$  is maximized if

$$(UA)_{0,\text{opt}}^{\mathsf{R}} = (UA)_{\mathrm{L,opt}} \tag{13}$$

while the total thermal conductance inventory of the refrigerator portion is constrained,

$$(UA)_{\rm R} = (UA)_0^{\rm R} + (UA)_{\rm L}.$$
 (14)

(a) 0.05 (b) 0.7  $\tau_{\rm L}$  = 0.9  $\tau_{L} = 0.9$ Q<sub>L,max</sub> UAT<sub>0</sub> 0.5 0.4 0.025 0.3 0.2 0.1 0-0.5 0.01 0.1 х

Fig. 3. (a) The maximization of the refrigeration load with respect to the conductance allocation ratio x. (b) The effect of x on the coefficient of performance.

By repeating the analysis presented in ref. [8], it can be shown that the maximum  $Q_L$  that corresponds to the optimization rule (13) is

$$Q_{\rm L,max} = \frac{1}{8} (UA)_{\rm R} T_0 \{ [(4\tilde{W} - \tau_{\rm L} + 1)^2 + 16\tau_{\rm L} \tilde{W}]^{1/2} - 4\tilde{W} + \tau_{\rm L} - 1 \}$$
(15)

where

$$\tau_{\rm L} = \frac{T_{\rm L}}{T_0} < 1 \tag{16}$$

$$\tilde{W} = \frac{W}{(UA)_{\rm R}T_0}.$$
(17)

We now return to the original problem formulated in Fig. 1, by coupling the optimized (P) and (R) portions of Fig. 2. This amounts to setting  $W = W_{\text{max}}$  in equations (15) and (17), for which  $W_{\text{max}}$  is given by equation (10). The total thermal conductance inventory UA of the entire installation, equation (6), can be written also in terms of the total inventories of the (P) and (R) portions,

$$UA = (UA)_{\rm P} + (UA)_{\rm R}$$
, (constant) (18)

or by introducing the fraction x

$$x = \frac{(UA)_{\rm P}}{UA}, \quad 1 - x = \frac{(UA)_{\rm R}}{UA}.$$
 (19)

After these substitutions, equation (15) becomes

$$\frac{Q_{\rm L,max}}{UAT_0} = \frac{1}{8} (1-x) \{ [(4\tilde{W}_{\rm max} - \tau_{\rm L} + 1)^2 + 16\tau_{\rm L}\tilde{W}_{\rm max}]^{1/2} - 4\tilde{W}_{\rm max} + \tau_{\rm L} - 1 \}$$
(20)

where

$$\tilde{W}_{\max} = \frac{x(\tau_{\rm H}^{1/2} - 1)}{4(1 - x)}.$$
(21)

The expression listed in equation (20) has been maximized numerically with respect to x, while holding the external temperature ratios  $\tau_{\rm H}$  and  $\tau_{\rm L}$  fixed. The procedure is illustrated in Fig. 3(a). It is interesting to point out that during this maximization the coefficient of performance COP =  $Q_{\rm L,max}/Q_{\rm H}$  does not exhibit a maximum with respect to x, Fig. 3(b). The reason is that  $\tau_{\rm H}$  is assumed fixed, the heat input  $Q_{\rm H}$  is limitless and the only restriction is size and allocation of the UA inventory.

The optimal allocation fraction  $x_{opt}(\tau_H, \tau_L)$  obtained based on Fig. 3(a) is shown in Fig. 4. The answer sought in connection with equation (6) is now complete: it is contained in Fig. 4 and equations (9) and (13). In conclusion, the optimal three-way allocation of *UA* between the generator, condenser and evaporator is

$$(UA)_{\rm H,opt} = \frac{1}{2} x_{\rm opt} UA \tag{22}$$

$$(UA)_{0,\text{opt}} = \frac{1}{2}UA \tag{23}$$

$$(UA)_{L,opt} = \frac{1}{2}(1 - x_{opt})UA.$$
 (24)

It is interesting that the optimal condenser always demands half of the total thermal conductance inventory, regardless of the temperature ratios  $T_{\rm H}/T_0$  and  $T_{\rm L}/T_0$ . Figure 4 shows that when  $T_{\rm L}/T_0$  is not much smaller than 1, the ratio  $x_{\rm opt}$  is equal to roughly  $\frac{1}{2}$ . This means that the optimal generator and evaporator conductances must have approximately the same size,

$$(UA)_{\mathrm{H,opt}} \simeq (UA)_{\mathrm{L,opt}} \simeq \frac{1}{4} UA.$$
 (25)

Figure 5 shows the maximum refrigeration rate that corresponds to the optimal allocation of the UA inventory (equations (22)–(24) and Fig. 4). The dimensionless group listed on the ordinate is a reminder that



Fig. 4. The optimal thermal conductance allocation ratio  $x = (UA)_P/UA$ .



Fig. 5. The maximum refrigeration rate of a heat driven refrigerator with fixed thermal conductance.

 $Q_{\rm L}$  has been maximized twice, first based on equation (13), and, second, by using  $x_{\rm opt}$ . The resulting  $Q_{\rm L}$  maximum increases monotonically with the heat source temperature  $\tau_{\rm H}$ , and decreases monotonically as the refrigeration load temperature  $\tau_{\rm L}$  decreases.

### 4. THE EFFECT OF THE COST OF THE HEAT INPUT

The optimization described so far was based on maximizing the refrigeration load  $Q_L$ . This choice is appropriate only when the cost of the heat input  $Q_H$ is negligible when compared with the economic value of the refrigeration effect  $Q_L$ . In general, however, the heat input is not available freely, and the relevant objective function to maximize is the profit

$$P = p_{\rm L}Q_{\rm L} - p_{\rm H}Q_{\rm H} \tag{26}$$

Here  $p_L$  and  $p_H$  are the known prices of the refrigeration load and the heat input. We are interested in dividing the total UA inventory of equation (6) such that P is maximized.

The maximization of P can be performed numerically, and our results are shown in Figs. 6(a)-(c). The problem statement consists of equations (1)-(6) and (26). We first non-dimensionalized these equations using equations (12) and (16) and

$$\tau_{\rm HC} = \frac{T_{\rm HC}}{T_0}, \quad \tau_{\rm 0C} = \frac{T_{\rm 0C}}{T_0}, \quad \tau_{\rm LC} = \frac{T_{\rm LC}}{T_0}$$
 (27)

$$\tilde{Q}_{\rm H} = \frac{Q_{\rm H}}{UAT_0}, \quad \tilde{Q}_0 = \frac{Q_0}{UAT_0}, \quad \tilde{Q}_{\rm L} = \frac{Q_{\rm L}}{UAT_0}.$$
 (28)

For brevity, we omit the resulting dimensionless equations, and note that equations (1)–(6) deliver  $\tilde{Q}_L$  as a function of five dimensionless numbers,  $\tilde{Q}_H$ ,  $\tau_H$ ,  $\tau_L$ , y and z, where

$$y = \frac{(UA)_{\rm H}}{UA}, \quad z = \frac{(UA)_{\rm L}}{UA}.$$
 (29)

Note also that according to the UA constraint (6), the equipment fraction allocated to the condenser is

$$\frac{(UA)_0}{UA} = 1 - y - z.$$
(30)



Fig. 6. The effect of the price ratio  $p_{\rm H}/p_{\rm L}$  on the optimal allocation of thermal conductance between the three heat exchangers.

In conclusion, the non-dimensionalized profit function

$$\tilde{P} = \frac{P}{p_{\rm L} U A T_0} = \tilde{Q}_{\rm L} - \frac{p_{\rm H}}{p_{\rm L}} \tilde{Q}_{\rm H}$$
(31)

depends on  $\tilde{Q}_{\rm H}$ ,  $\tau_{\rm L}$ , y, z and  $p_{\rm H}/p_{\rm L}$ . We maximized  $\tilde{P}$  numerically by varying y and z while holding  $\tilde{Q}_{\rm H}$ ,  $\tau_{\rm H}$ ,  $\tau_{\rm L}$  and  $p_{\rm H}/p_{\rm L}$  constant. The pair of optimal values  $(y_{\rm opt}, z_{\rm opt})$  that maximizes  $\tilde{P}$  is reported in Figs. 6(a)–(c) for several combinations of  $\tilde{Q}_{\rm H}$ ,  $\tau_{\rm H}$ ,  $\tau_{\rm L}$  and  $p_{\rm H}/p_{\rm L}$ . Note that the limit  $p_{\rm H}/p_{\rm L} = 0$  represents the  $Q_{\rm L}$  maximization results described in the preceding section.

In ejector refrigeration cycle applications the source temperature  $T_{\rm H}$  varies between 80 °C and 130 °C, and the refrigeration load is extracted from temperatures



Fig. 7. Refrigerator driven by heat transfer from a solar collector.

 $T_{\rm L}$  ranging from -15 °C to 5 °C [11]. These ranges are represented approximately by  $\tau_{\rm H} = 1.3$  and  $\tau_{\rm L} = 0.9$ , which have been used to develop the numerical results shown in Fig. 6(a). The graph shows  $y_{\rm opt}$  and  $z_{\rm opt}$  in a way that illustrates the splitting of the total thermal conductance (an amount equal to 1 on the ordinate) between the generator, condenser and evaporator.

The most striking feature is that the condenser demands half of the total thermal conductance, regardless of the price ratio  $p_{\rm H}/p_{\rm L}$  and the external temperature levels ( $\tau_{\rm H}$ ,  $\tau_{\rm L}$ ). In other words, the conclusion reached in (23) is more general than in the model of Section 3. In each frame of Fig. 6 we see that  $(UA)_{\rm L,opt}$  increases at the expense of  $(UA)_{\rm H,opt}$  as the price ratio  $p_{\rm H}/p_{\rm L}$  increases.

Comparing Figs. 6(a) and (b) we learn that when  $\tau_{\rm H}$  increases, the evaporator conductance must be increased, again, at the expense of the generator conductance. Figures 6(b) and (c) show that as  $\tau_{\rm L}$  decreases from 0.9 to 0.8 it has a negligible effect on the conductance allocation fractions  $y_{\rm opt}$  and  $z_{\rm opt}$ .

#### 5. THE OPTIMAL COUPLING BETWEEN THE REFRIGERATOR AND A SOLAR COLLECTOR

Consider now the class of applications where the heat input  $Q_{\rm H}$  is provided at the temperature level  $T_{\rm H}$  by a flat plate solar collector, Fig. 7. The relation between  $Q_{\rm H}$  and the collector temperature  $T_{\rm H}$  can be expressed as

$$Q_{\rm H} = A_{\rm C} G_{\rm T} [a - b(T_{\rm H} - T_0)]$$
(32)

where  $A_c$  is the collector area,  $G_T$  is the irradiance at the collector surface, and *a* and *b* are two constants that can be calculated as shown by Sokolov and Hershgal [12]. Equation (32) represents a collector with partial heat loss to the ambient. The group  $[a-b(T_{\rm H}-T_{\rm 0})]$  is known as the collector efficiency, and

$$T_{\rm st} = T_0 + \frac{a}{b} \tag{33}$$

is the stagnation (i.e. the ceiling) temperature of the collector. When  $T_{\rm H} = T_{\rm st}$  the heat input  $Q_{\rm H}$  is zero.

Sokolov and Hershgal [12] demonstrated that when the collector and heat exchanger are specified, there exists an optimal collector temperature for maximum refrigeration effect, i.e. an optimal *coupling* between the solar collector and the refrigerator. In this section we examine how this coupling is affected by the sizes of the heat exchangers. We begin with the observation that equation (32) replaces equation (1) of the earlier model, and that the collector  $(A_C, G_T)$  replaces the generator  $(UA)_H$ . The solar-driven refrigerator continues to be described by the right side of Fig. 1, for which  $Q_H$  is given by equation (32), and the total UAinventory is to be shared by the condenser and the evaporator,

$$UA = (UA)_0 + (UA)_1.$$
(34)

The operation of the refrigerator is governed by equations (2)–(5), (32) and (34). These can be restated in dimensionless terms by using  $\tau_{0C}$ ,  $\tau_{LC}$ ,  $\tilde{Q}_{H}$ ,  $\tilde{Q}_{0}$  and  $\tilde{Q}_{L}$  defined in equations (27) and (28), and the additional parameters

$$\tau_{\rm st} = \frac{T_{\rm st}}{T_0} \quad B = \frac{bA_{\rm C}G_{\rm T}}{UA}.$$
 (35)

We continue to use the evaporator conductance allocation ratio  $z = (UA)_L/UA$ , and note that this time  $(UA)_0/UA = 1-z$ . The *B* parameter describes the size of the collector relative to the cumulative size of the condenser and the evaporator. By solving equations



Fig. 8. The optimal collector temperature  $(\tau_{H,opt})$  and thermal conductance allocation ratio  $(z_{opt})$ . (a) The effect of the collector size (B); (b) the effect of the stagnation temperature  $(\tau_{st})$ ; (c) the effect of the refrigeration temperature  $(\tau_{r})$ .

(2)–(5) and (32) numerically, we were able to determine  $\tilde{Q}_{L}$  for a given set of values for B,  $\tau_{\rm H}$ ,  $\tau_{\rm L}$ ,  $\tau_{\rm st}$  and z. We varied  $\tau_{\rm H}$  and z until we located the pair ( $\tau_{\rm H,opt}$ ,  $z_{\rm opt}$ ) that maximizes  $\tilde{Q}_{\rm L}$ . The resulting  $\tau_{\rm H,opt}$  and  $z_{\rm opt}$  values are functions of three parameters, B,  $\tau_{\rm st}$  and  $\tau_{\rm L}$ .

Figure 8(a) shows the effect of the collector size (B) on the optimal collector temperature and thermal conductance allocation ratio. We see that the collector temperature increases only slightly as the B parameter increases by a factor of 10 (from 0.1 to 1). The B effect on  $z_{opt}$  is even weaker: in the B range of Fig. 8,  $z_{opt}$  is such that the optimal evaporator conductance  $(UA)_L$  is roughly one third of the total conductance  $(UA)_0$ . This

conclusion agrees approximately with the relative size  $(UA)_{L,opt}/(UA)_{0,opt} \simeq \frac{1}{2}$  described by equations (23) and (24).

The effect of the stagnation temperature is illustrated in Fig. 8(b). Both  $\tau_{H,opt}$  and  $z_{opt}$  increase as  $\tau_{st}$ increases. Worth noting is that  $\tau_{H,opt}$  is consistently greater than  $\tau_{st}^{1/2}$ , i.e. greater than the optimal collector temperature determined in ref. [16] for a collector coupled to a power cycle that has no heat transfer irreversibilities. Finally, Fig. 8(c) shows that the refrigeration temperature  $\tau_L$  has an insignificant effect on the optimal results determined in this section.

#### 6. CONCLUSION

In this paper we presented the thermodynamic optimization of heat driven refrigeration plants. This was based on a model (Fig. 1) that accounted for the irreversibility of the plant and the finiteness of the heat exchanger inventory (total thermal conductance). We determined the operating regime for maximum refrigeration effect, and saw how the optimal performance is affected by the extreme temperature levels of the refrigeration plant (Fig. 5).

From a practical standpoint, the most important conclusion is that the maximum refrigeration regime requires that the thermal conductance be allocated in a certain way between the three heat exchangers (Fig. 4). The allocation of the thermal conductance is influenced to some extent by the relative price of the heat source (Fig. 6). The optimal thermal conductance of the ambient-temperature heat exchanger is half of the total supply, and is independent of the relative price of the heat source. The example of the solar driven refrigerator (Section 5) showed that the basic thermodynamic optimization principles developed in the first part of the paper can be used to optimize actual refrigeration plants that are driven by heat transfer.

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